

Consider the region defined by  $y \leq 4 - x^2$  and  $y \geq -3x$ .

$$x = \pm \sqrt{4-y} \quad x = -\frac{1}{3}y$$

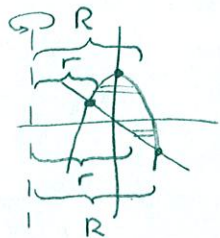
ALL ITEMS

SCORE: \_\_\_\_ / 14 PTS

- [a] If the region is revolved around the line  $x = -6$ , write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid

① POINT EACH

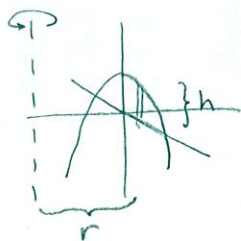
- [i] using the disk or washer method (**NOTE: You do NOT need to simplify your integrand.**)



$$\pi \int_{-12}^3 \left( (\sqrt{4-y} + 6)^2 - \left(-\frac{1}{3}y + 6\right)^2 \right) dy + \pi \int_3^4 \left( (\sqrt{4-y} + 6)^2 - (-\sqrt{4-y} + 6)^2 \right) dy$$

$$\begin{aligned} 4 - x^2 &= -3x \\ 0 &= x^2 - 3x - 4 \\ 0 &= (x-4)(x+1) \\ x &= -1, 4 \\ &\downarrow \quad \downarrow \\ y &= 3 \quad y = -12 \end{aligned}$$

- [ii] using the shell method (**NOTE: You do NOT need to simplify your integrand.**)



$$2\pi \int_{-1}^4 (x+6)(4-x^2+3x) dx$$

- [b] Suppose the region is the base of a solid. Cross sections perpendicular to the  $x$ -axis are isosceles right triangles with one leg in the base. Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid.



$$\frac{1}{2} \int_{-1}^4 (4-x^2+3x)^2 dx$$



The region bounded by  $y = x^2 - 9$ ,  $y = 2(x - 5)$  and  $y = 0$  is revolved around the line  $y = 10$ .

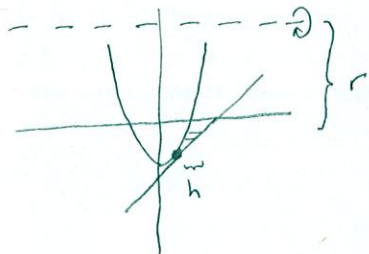
SCORE: \_\_\_\_ / 8 PTS

Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid **using as few integrals as possible**.

$$x^2 - 9 = 2x - 10$$

$$\underline{x^2 - 2x + 1 = 0} \quad \textcircled{1}$$

$$x = 1 \rightarrow y = -8$$



$$\textcircled{1} \underbrace{2\pi}_{\textcircled{1}} \int_{\textcircled{1}-8}^{\textcircled{1}0} \underbrace{(10-y)}_{\textcircled{1}} \underbrace{\left(\frac{y}{2} + 5 - \sqrt{y+9}\right)}_{\textcircled{3}} dy$$

Find the area bounded by the curves  $y = 4x^3$  and  $y = 16x^2 - 12x$ .

SCORE: \_\_\_\_ / 8 PTS

**NOTE: Your final answer must be a number, not an integral nor sum of integrals.**

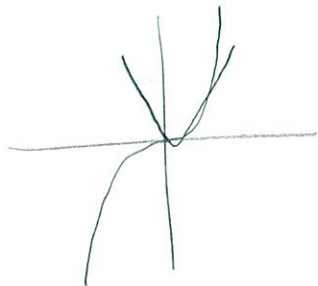
$$4x^3 = 16x^2 - 12x$$

$$4x^3 - 16x^2 + 12x = 0$$

$$4x(x^2 - 4x + 3) = 0$$

$$4x(x-1)(x-3) = 0$$

$$x = 0, 1, 3$$



$$\begin{aligned} & \int_0^1 (4x^3 - (16x^2 - 12x)) dx + \int_1^3 (16x^2 - 12x - 4x^3) dx \\ &= \left( x^4 - \frac{16}{3}x^3 + 6x^2 \right) \Big|_0^1 + \left( -x^4 + \frac{16}{3}x^3 - 6x^2 \right) \Big|_1^3 \\ &= \left( 1 - \frac{16}{3} + 6 \right) + (-81 + 144 - 54) - \left( -1 + \frac{16}{3} - 6 \right) \\ &= 7 - 5\frac{1}{3} + 9 + 7 - 5\frac{1}{3} \\ &= 12\frac{1}{3} \end{aligned}$$